# Artificial neural networks in civil engineering: another five years of research in Poland

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This state-of-the-art-paper is a resumé of research activity of a non-formal Research Group on Artificial Neural Networks (RGANN) applications in Civil Engineering (CE). RGANN has been working at the Cracow University of Technology, Poland, since 1996 under the supervision of the author of this paper. Ten years 1996–2005 of the research and teaching activity of RGANN was reported in paper [61]. The present paper briefly reports on the activities originated in the ten year period and their continuation after 2005. The main attention is focused on new research carried out in the five year period 2006–2011. The paper discusses some selected problems which are included in fourteen supplementary papers, marked in references of these papers as published in this CAMES Special Issue.

The attention is focused on: Hybrid Computational Systems, development of modifications of ANNs and methods of their learning, Bayesian neural networks and Bayesian inference methods, damage identification in CE structures, structure health monitoring, applications of ANNs in mechanics of structures and materials, joining of ANNs with measurements on laboratory models and real structures, development of new non-destructive measurement methods, applications of ANNs in health structure monitoring and repair, applications of ANNs in geotechnics and geodesy. The paper is based on the supplementary papers which were presented at the Special Session on Applications of ANNs at the 57th Polish Civil Engineering Conference in Krynica, 2011, see [74].

Keywords: Civil Engineering Artificial (CE), Neural Network (ANN), Standard NN (SNN), Multi-Layred Perceptron (MLP), Fuzzy Weight NN (FWNN), Kalman Filtering (KF), Bayesian NN (BNN), True-BNN (TBNN), Semi-Bayesian NN (SBNN), Gaussian Process (GP), Recurrent Cascade NN (RCNN), Principle Component Analysis (PCA), Hybrid Computational System (HCS), Finite Element Method (MES), Empirical Data (EMP), Hybrid Monte Carlo Method (HMCM), Hybrid Updated Algorithm (HUA), Neural Material Model (NMM), Structural Health Monitoring (SHM).

# **1. INTRODUCTION**

In years 1996 and 1997, an informal research group and a standing seminar on applications of artificial neural networks in civil engineering, called RGANN and SemANN for short, started at the Institute of Computer Methods in Civil Engineering (now Institute of Computational Civil Engineering) of the Cracow University of Technology. The RGANN and SemANN gathered participants of seven South Poland universities of technology under the supervision of the author of this paper. Owing to the participants' enthusiasm, besides research and teaching, also publishing of new scientific results and their presentation at many scientific conferences and congresses took place.

A state-of-the-art paper [61] was published in 2006, in which the main results obtained in ten years period 1996–2005 of RGANN and SemANN were discussed. The attention was focused on five areas: i) ANN as a new independent computational tool for the analysis of civil engineering (CE) problems, ii) analysis of various topics of CE applying ANNs, iii) hybrid systems with interaction of two components, i.e. finite element method and ANNs for the analysis of large problems in CE (FEM/ANN in CE), iv) modifications of standard NN architectures and development of methods for their learning, v) teaching and promotion of ANNs. In [61] six selected study cases were quoted to illustrate new results obtained in years 1996–2005.

Besides the above listed problems, some new approaches were developed in the first ten years of activities, which have then been continued over another five year period 2006–2011. This concerns especially topics of Ph.D. Theses and D.Sc. monographs (Polish upper scientific level dissertations, called "habilitations"). They were related to various applications of ANNs to the analysis of direct and inverse problems and development of hybrid computational systems. At the end of the first research period the ANNs were also applied in the analysis of structure health prediction and applications of neural networks in selected problems of geotechnics and geodesy.

# 2. GENERAL REMARKS ON RESEARCH IN YEARS 2006–2011

The activities of RGANN and SemANN have been slightly reorganized since their supervisor, prof. Z. Waszczyszyn after his retirement, accepted the position of full professor at the Rzeszów University of Technology. That is why he was also able to cooperate closer with Prof. L. Ziemianski's research group. SemANN has continued its activity at the Cracow University of Technology. The present paper is devoted to the next five years 2006–2011 of RGANN and SemANN activities. The paper was written in an extended form as the fist one in a cycle of papers presented at the Special Session on ANNs in CE at the 57th Polish Civil Engineering Conference in Krynica Poland in 18–22 September, 2011.

The paper deals with general activities that were proceeded by RGANN and SemANN in the last ten years. It is supplemented by fourteen papers published in the presented CAMES Special Issue following the papers delivered at the 57th Krynica Conference. That is why, inlike to paper [61], besides references to the above mentioned fourteen papers, only some distinguished basic approaches, developed in recent years, are discussed in short.

The activities after 2005 can be roughly divided into two groups:

- 1. Continuation of research, teaching and promotion of ANN applications in CE, which started in the previous ten year period.
- 2. New research, teaching and promotion of ANNs, which have been developed after 2005.

# 3. CONTINUATION OF ACTIVITIES ORIGINATED IN PREVIOUS TEN YEARS 1996–2005

# 3.1. D.Sc.E. monographs and Ph.D. Theses

Two D.Sc.E. monographs, written by Pabisek [41] and Sulewska [53] were published after 2005. Eight Ph.D. Theses, quoted here in chronological order, were written and "defended" after 2005 by Mrówczyńska [34], Kaczmarczyk [14], Krok [19], Buda-Ożóg [4], Jakubek [12], Nazarko [38], Wojciechowski [70] and Borowiec [3].

#### **3.2.** Analysis of regression problems

Without any doubt, ANNs appeared to be an efficient new tool in the analyses of classification and regression problems. From the very beginning, the members of RGANN and SemANN focused their attention on the analysis of regression problems in CE.

The regression problems are related to mapping of input vectors  $\mathbf{x}^p = \{x_j^p\}_{j=1}^D$  for  $p = 1, \ldots, P$  patterns onto output vector functions  $\mathbf{y}(\mathbf{x}; \mathbf{w}) = \{y_m(x, w)\}_{m=1}^M$ , where:  $\mathbf{w} = \{w_i\}_{i=1}^W$  – weight vector of ANN model.

ANNs can be formulated for the analysis of various regression problems. This can de performed specifying the input/output data according to Paez classification [47], shown in Table 1 for mechanical systems (MS).

| Problems  | Inputs  | Outputs                         |
|---|---|---------------------------------|
| A. Mechanical system (MS) response<br>simulation  | Excitation variables and system parameters                | Response variables              |
| B. MS excitation simulation (identifica-<br>tion) | Response variables and system parameters                  | Excitation variables            |
| C. MS parameter identification                    | MS excitation and response variables                      | Parameters of MS                |
| D. MS excitation and/or response as-<br>sessment  | All relevant measures of system conditions to be assessed | Assessment of system conditions |

 Table 1. Mechanical system problems and relevant data.

Simulation problem A refers to problems called in mathematics *direct problems*. Problems B and C are related to *inverse problems*. In engineering problems, the inverse analysis is connected especially with design problems, which are frequently analyzed as a sequence of simulations.

As was discussed in the state-of-the-art-papers [68, 69], inverse problems B (called external identification) and problem C (called internal or material model identification) can be efficiently analyzed by the standard neural network (SNN). In these networks the Least Square error  $E_{\rm LS}$  (**w**) is adopted:

$$E_{\rm LS}(\mathbf{w}) = \frac{1}{2} \sum_{p=1}^{P} \{t^p - y(\mathbf{x}^p; \mathbf{w})\}^2,\tag{1}$$

where for the sake of clarity only the single output network with a target data set  $\{t^p\}_{p=1}^P$  is applied.

SSNs suitable for the regression analysis are, first of all, the feed-forward neural networks either of layered structure or related to the Radial Basis Function Neural Networks (RBFNs). The layered NN called in [11] Multi-Layred Perceptron (MLP) were favourably applied in RGANN and SemANN research. After 2005 the interest in RBFNs started, mainly because of their applications in Bayesian computational systems, see Subsec. 4.1.

A great deal of attention was also paid to the application of Paez' classification and formulation of appropriate input/output data in the analysis of complex problems. This can need the formulation of hybrid computational systems (HCS). Such systems have been developed since the beginning of the previous ten year research period, see [43, 61] and it has been extensively continued after 2005.

# 3.3. Hybrid computational systems

# 3.3.1. General remarks

A great deal of attention has been paid to hybrid computational systems (HCSs). Such systems are composed of components of different and reciprocal features. Let us consider the component, corresponding to the finite element method (FEM), which is extremely efficient in the analysis of direct problems. In case the second component is an ANN, then we can compose a HCS, whose components can interact with each other in the analysis of complex problems. The component FEM is based on hard computing, vs. ANN supported on neurocomputing. Such features open the door for increasing the numerical efficiency of the FEM&ANN system of co-called low degree of component fusing, cf. [41]. This system is usually formulated of separate parts corresponding to FEM and ANN, which enable the application of 'off line' computational technique. In case the ANN is inside FEM, then the 'on line' technique can increase significantly the numerical efficiency. Such a system will be written in the form FEM/ANN, corresponding to a high degree of component fusion, cf. e.g. [41].

In years 2006–2011 a special attention was focused on three component HCS identification problems, related to Paez' cases B and C (cf. Table 1). In full identification problems, besides computational components FEM and ANN, component EMP has to be added. It corresponds to empirical data related to a set of measurements on different levels of real structures or in laboratory tests. In general HCS of different structures should fit well the analyzed engineering problems.

HCS components can serve different purposes. In case of full identification, the EMP component can be formulated as pseudo-empirical data p-EMP, computed usually as a computer simulation of real measurements. Such simulations can be very useful to start with a HCS before tests or even to plan tests on real structures or laboratory models. In case of internal identification (case C in Table 1) the component EMP can be exploited for calibration of internal parameters, e.g. values of material characteristics.

In case we have no measurements on physical material models we can simulate p-EMP by theoretical models well certified by tests on laboratory specimens or structural elements. Models of materials, solids and soils, formulated in the theory of elasticity or plasticity, are a good example.

In the research carried out in 2006–2011 two types of HCS were developed, i.e. FEM&SNN&EMP and FEM/SNN/EMP. The first system is of low degree of component fusion vs. FEM/SNN/EMP which is of high degree of fusion, cf. [41].

# 3.3.2. Problems analyzed by HCS

Hybrid Monte Carlo Method in reliability analysis of structures. In her Ph.D. Thesis, "defended" in 2005, [16], J. Kaliszuk analyzed a number of problems related to the reliability analysis of simple CE structures. Two of them, i.e. load carrying capacity of a steel girder beam and circular panels were presented at scientific conferences after 2005. The extended supplementary paper [17] discusses in detail the application of the Hybrid Monte Carlo Method (HMCM) proposed in [46] to the analysis of the above mentioned problems. Two HCMs were formulated in the form FEM/SNN and FEM/SNN/EMP.

The simulation regression algorithm was supported on HMCM, in which FEM was based on the geometric material nonlinear models of structures and materials in order to compute the ultimate load parameter  $\lambda_{ult}(\mathbf{X})$ , corresponding to overall buckling of structure, where:  $\mathbf{X}$  – random input parameters related to structure and material. A small set of computed FEM solutions  $\{\mathbf{X}^p, t_{ult}^p\}_{p=1}^P$  was adopted for the design and training of small SSNs, which were applied to computations of Monte Carlo trial as the mapping  $\mathbf{X} \to \lambda_{ult}$ . In the reliability analysis of a steel girder and cylindrical panels the FEM code COSMOS-D was applied. In the case of panels the code was updated by means of tests on laboratory models, see [18]. It was proved that such a hybrid approach enables the computation of the structure reliability curves in CPU times significantly lower than those that could be obtained hypothetically by the application of the FE code only.

**Updating of simple structures.** In the Ph.D. Thesis [31], written by B. Miller, a Hybrid Updating Algorithm (HUA) was formulated as the HCS of low degree of component fusion FEM&ANN&EMP. The HUA was composed of five stages, cf. supplementary paper [32].

The HUA starts from a direct analysis, similarly to the approach mentioned above in the Hybrid Monte Carlo Method (HMC). The mapping  $\boldsymbol{\alpha} \to \mathbf{r}$  is carried out as a direct analysis by means of FEM, where:  $\boldsymbol{\alpha}$  – vector of control parameters,  $\mathbf{r}$  – vector of structure responses. Then an artificial noise is added to the response set of patterns  $\{\mathbf{r}^{p'}\}_{p=1}^{P'}$ , which becomes the input set for the inverse analysis  $\{\mathbf{r}^{p'}\}_{p=1}^{P'} \to \boldsymbol{\alpha}_{ANN}$ . Having measured responses  $\{\mathbf{r}^m\}_{m=1}^M$ , it is possible to calibrate the control parameters  $\boldsymbol{\alpha}_{ident} = \boldsymbol{\alpha}_{ANN}(\{\mathbf{r}^m\})$ . The vector of identified control parameters  $\boldsymbol{\alpha}_{ident}$  is substituted to the FEM code in order to have an updated program FEM<sub>upd</sub>.

The above mentioned hybrid updated algorithm was developed by Miller in [31–33]. He discussed two problems, corresponding to laboratory simple models, made of aluminum alloys, i.e. a thin-walled suspended beam and a two-story plane frame. In the former problem two internal parameters,

corresponding to an additional mass density  $\rho_L^e$  and shear correction factor  $k_s$  were adopted as control parameters. They were identified by means of HUA also for damaged beams. The other example was related to the updating of a FEM<sub>upd</sub> code to predict correctly some of the first eigenfrequencies of the laboratory tested frame. Stiffness of additional springs and the length of an equivalent supporting FEs were adopted as control parameters. It was numerically proved that the application of the proposed algorithm HUA gave computational predictions very close to that measured on the laboratory tests.

Identification of equivalent materials in real structures and solids. In her DScE. monograph [41], E. Pabisek discussed various HCS. In recent five years she has focused on development of FEM/SNN/p-Emp systems of a very high degree of component fusion. The main research goal was to formulate HCS for the identification of equivalent material models in solids, called Neural Material Models (NMMs). The models are made of unknown 'a priori' material (e.g. material of real structures), corresponding to measurement data completed in the component EMP or p-EMP. The main attention was paid to development of the Ghaboussi's Autoprogressive Method (AM), see [7, 8, 10].

The main idea of AM lies in 'on line' computations of patterns for SNN learning, training and updating, as well as fulfilling the compatibility conditions by means of measurement (or pseudomeasurement) data, completed in the EMP component. In papers [42, 43] two algorithms of AM were discussed. The first one, called algorithm A, fully corresponds to the Ghaboussi's autoregressive method, proposed in [7], then discussed in many papers, cf. references in [8]. This algorithm depends on updating the patterns and training NMM at each step n of the load parameter increment  $_n\Delta\lambda$ . The other algorithm, B (called in [42, 43] cumulative algorithm) corresponds to the learning of NMM at the last load increment, i.e. once for each load cycle c. This algorithm was introduced by Shin and Pande in papers [51, 52]. Both algorithms have their advantages and disadvantages, see discussion in [42, 41].

In Pabisek's D.Sc. monograph [41] some identification plane stress BV problems were analyzed. The p-EMP equilibrium paths were simulated for materials depending on the  $J_2$  stress deviator (mainly steel and aluminum alloys elements). For such materials algorithm A seemed to be superior. In the supplementary paper [44] the identification of sand soil was discussed. The soil plasticity model, proposed by Drucker and Prager, was adopted to obtain pseudo-empirical data p-EMP. This material is pressure dependent, so besides the stress deviator invariant  $J_2$  also the stress tensor invariant  $I_1$  has to be taken into account. After a number of computational experiments it was stated that in the HCS identification algorithm B was more advantageous than algorithm A. An interesting new result was presented in [44]. This concerns a possibility of the NMM identification on a sub-domain of the investigated BV plain stress problem, related to the superficial foundation on a semi-plane.

# 3.4. Development of ANNs and learning methods

# 3.4.1. Fuzzy Weight Neural Network

Before 2005 a fuzzy neural network called FWNN (Fuzzy Weight Neural Network) was developed by Pabisek, Jakubek and Waszczyszyn, see [45]. Membership functions of FWNN were formulated by the MLP network training, separately for each learning pattern and then the interval arithmetic was applied to process crisp or fuzzy data.

FWNNs were applied by Jakubek in her Ph.D. Thesis [12] for selected problems of material and structural mechanics related to evidence of laboratory tests. The following three problems are discussed in the supplementary paper [13]: 1) simulation and parametric identification of the strength and number of fatigue failure cycles of normal concrete specimens, 2) strength prediction of high performance concrete mixtures, 3) identification of buckling loads of eccentrically loaded reinforced-concrete columns.

# 3.4.2. Kalman neural filtering

Kalman filters (KF) are widely applied in the analysis of dynamic linear processes. In A. Krok's Ph.D. Thesis [19] KF extended and decoupled equations were used in the nonlinear analysis of material and structure problems with sequential data. These equations were adopted as a base for formulating a new method for MLPs learning.

From among many problems discussed in [19] the problem of simulation of hysteretic loops for twelve tests on compressed concrete specimens are discussed in the supplementary paper [20]. Additionally, the application of the Gaussian Process and model calibrated by Genetic Algorithms is discussed. The best approximation was obtained by means of Bayesian methods.

# 3.4.3. Replicator neural network

A great number of inputs can influence significantly the size of designed SNNs (*size* means here the number of SNN weights). This problem can be of importance for small data sets. That is why the problem of reduction of the input space dimensionality D without losing relevant information, might be important for SNNs formulation.

One of possible methods of reduction of D is the application of the autoassociated MLP, called replicator, see [11]. This approach was introduced into structural dynamics by Lin and Ghaboussi, see references in [8, 28]. The replicator is a two layer autoassociated MLP of structure D-d-D, where: D – number of inputs/outputs, d < D – number of hidden neurons, corresponding to the number of reduced inputs, see Fig. 1 taken from [68]. The data compression and decompression are also shown in Fig. 1. It corresponds to the mapping  $\mathbf{x}_{(D\times 1)} \to \widetilde{\mathbf{x}}_{(d\times 1)} \to \widehat{\mathbf{x}} = \mathbf{x} - \boldsymbol{\varepsilon}$ , where  $\widetilde{\mathbf{x}} \in \mathcal{R}^d$ – compressed input vector,  $\boldsymbol{\varepsilon}$  – error of the replicator learning process.

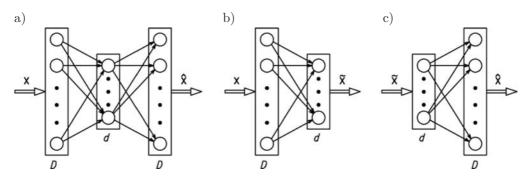


Fig. 1. a) MLP as a replicator, b,c) splitting of replicator into compressor and decompressor.

Before 2005 the replicator was used in the buckling analysis of cylindrical shells, see [63] and in the analysis of several structural dynamics problems, cf. [24]. After 2005 the replicator was applied in damage classification, see [40]. Instead of replicators the Principle Component Analysis was widely applied in the recent research period 2006–2011, see Subsubsec. 4.2.1.

# 3.5. SNNs for analysis of building vibrations

Since the very beginning of research activity of RGANN and SemANN, the problems of mineinduced building vibrations attracted a great deal of attention of participants of the two groups. This was related to the research activity of Kuźniar, reflected in her D.Sc.E. monograph [23], Chapter XVI in book [26, 27] and supplementary paper [24].

The following four groups of problems were discussed in [24]: 1) prediction of building fundamental period of natural vibrations, 2) mapping of mining tremors into response spectra from ground vibrations, 3) soil-structure interaction analysis, 4) simulation of building responses to paraseismic excitations. A deal of attention was focused on the input data preprocessing and application of replicators, especially before 2006.

# 3.6. Damage detection in structure health analysis and laboratory tests

A great deal of efforts was spent on the identification of damage in structural elements and solids.

Starting from Łakota's D.Sc. monograph [29] written in 1999, the book by Ziemiański [72] and many papers by him and his coworkers from the Rzeszów University of Technology can be quoted, see references in the state-of-the-art-papers [69, 73]. The investigated problems were related to identification of loads, material parameters, damage localization in structure elements and simple structures (beams and plane frames).

Two basic approaches were adopted. The first one was based on adoption of measurements or computer simulations of eigenfrequencies as structures responses, see references in [68, 69, 72, 73]. The other approach was the analysis of transmission and reflections of ultrasonic elastic waves in elastic strips and continua [38–40, 68, 72]. This approach was extensively developed in recent years in order to formulate new non-destructive methods for diagnosis and monitoring of materials and structures in CE, in the frame of areas called Structure Health Analysis and Monitoring (SHA&M).

Parallel to the development of theoretical foundations tests on laboratory models were continuously carried out. Lakota's experimental activity was continued by Miller, Piątkowski, Nazarko, Buda-Ożóg and Borowicz, see their Ph.D. Theses [3, 4, 31, 38, 49] and supplementary papers [32, 40].

After 2005 P. Nazarko started with his research directed to formulation of tests on nondestructive methods for the diagnosis of structural members damage. His Ph.D. Thesis was "defended" 'cum laude' and published as monograph [39]. The analysis of patterns carried out is based on transmission of waves in elastic strips. Structural Health Monitoring is discussed in the supplementary paper [40] by Nazarko and Ziemiański. A two stage hybrid approach ANN&EMP was discussed therein. The damage measurements were classified by means of a replicator and then MLP was applied to the identification of damage location and estimation of its geometrical parameters.

# 4. NEW RESEARCH AFTER 2005

#### 4.1. Bayesian Neural Networks

In years 2005–2006 M. Słoński turned his attention to the books written by Bishop [1, 2], devoted to the Bayesian Neural networks (BNNs). They are in fact a kind of refined systems, in which Bayesian inference was introduced into ANNs interface. Standard NNs can be obtained as a special case of BNNs.

# 4.1.1. Bayesian inference

Bayesian inference is based on Bayes' theorem, see [1, 2, 67]:

$$p(\mathbf{w}|\mathbf{t},\alpha,\beta) = \frac{p(\mathbf{t}|\mathbf{w},\beta)p(\mathbf{w}|\alpha)}{p(\mathbf{t}|\alpha,\beta)},\tag{2}$$

which can be verbalized: *posterior* = *likelihood* × *prior/evidence*. The evidence has the form of integral expressing the marginalization (elimination) of the random weight function  $\mathbf{w} \in \mathcal{R}^W$ :

$$p(\mathbf{t}|\alpha,\beta) = \int_{\mathcal{R}^W} p(\mathbf{t}|\mathbf{w},\beta)p(\mathbf{w}|\alpha)d\mathbf{w}.$$
(3)

The evidence (called also the marginal likelihood) can be computed numerically as the integral over all weight vector components in the weight space  $\mathcal{R}^W$ . In formulas (2) and (3) conditional probabilities p are applied, which can be verbalized. For instance, p ( $\mathbf{w}|A$ ) means "probability  $\mathbf{w}$ given A". In other words, this means that the random vectors  $\mathbf{w}$  and  $\mathbf{t}$  in (2) are conditioned by the target vector  $\mathbf{t}$  and hyperparameters  $\alpha$  and  $\beta$  in (3).

The Bayesian inference corresponds to an improvement of prior by means of additionally conditioned data. In case of Bayes' theorem of form (2) the probability of posterior for the weight vector  $\mathbf{w}$  is conditioned not only by information concerning the prior (given hyperparameter  $\alpha$ conjugated with  $\mathbf{w}$ ) but also by the target vector  $\mathbf{t}$  and hyperparameter  $\beta$  conjugated with the output data set.

# 4.1.2. Bayesian networks and their approximations

The computation of integral in evidence (3) is analytically intractable so the main problem of application of the Bayes' theorem lies in numerical computation of the integral (3). This corresponds to the marginalization (elimination) of the weight function  $\mathbf{w}$ .

The numerical integration procedures are computationally very costly (they need a great number of operations) in cases when the weight space  $\mathcal{R}^W$  is multidimensional. That is why, besides socalled True Bayesian NNS (TBNNs), in which numerical integration is carried out, a number of simplified approximations of BNN were developed, see [2, 67]. From among them the Semi Bayesian NNs (SBNNs) and Gauss Process in regression (GP) networks are particularly worth of attention.

#### 4.1.3. Semi Bayesian Neural Networks and Bayesian methods

These networks are of the standard neural networks structure, but the extended network error function  $F(\mathbf{w}; \alpha, \beta)$  is applied. It contains the terms corresponding to the Least Squared error  $E_{\text{LS}}$  and regularization penalty term  $E_{\text{W}}$ , weighted by the hyperparameters  $\alpha$  and  $\beta$ :

$$F(\mathbf{w};\alpha,\beta) = \alpha E_{\rm LS}(\mathbf{w}) + \beta E_{\rm W} \equiv \frac{\alpha}{2} \sum_{p=1}^{P} \{t^p - y(\mathbf{x}^p;\mathbf{w})\}^2 + \frac{\beta}{2} \mathbf{w}^{\rm T} \mathbf{w},\tag{4}$$

where for the sake of clarity only the single output network with a target set  $\{t^p\}_{p=1}^P$  is applied.

In what follows only the network of MLP structures with the extended error function (4) are applied in SBNN. Such a network is marked as SSN/MAP networks in [67], with the acronym corresponding to the Bayesian methods, see below.

# Computation of number of neurons in hidden layer of SBNN and hyperparameters

The main feature of SBNN is that it is designed and the hyperparameters are learned by means of Bayesian methods basing on the MML (Maximum Marginal Likelihood). MML is briefly described in the Appendix to paper [21] written by Kłos, Sulewska and Waszczyszyn.

Marginal likelihood is defined as the denominator in Bayes' theorem, corresponding to the integral (3). The MML criterion can be consistently expressed as:

$$\max_{\mathbf{w}} \ln ML \equiv \max_{\mathbf{w}} p(\mathbf{t}|\alpha,\beta) \to H_{\text{opt}},\tag{5}$$

written for the single hidden layer composed of H neurons. Appropriate formulas can be found in the books [1, 2, 67] and in the Appendix of the supplementary paper [21].

A great advantage of criterion MML is that  $H_{opt}$  can be computed having only the learning set of patterns. This means that this criterion can be used instead of the commonly used cross-validation

method but without an additional validation set of patterns. This was emphasized by Waszczyszyn and Słoński in papers [65, 66] written in Polish and English.

It is worth mentioning that the criterion MML has been known in literature devoted to BNNs since about 1992, cf. [30], but it has not been applied to the design of non-Bayesian NNs.

Maximizing the function  $\ln ML = p(\mathbf{t}|\alpha,\beta)$  with respect to hyperparameters leads to the following relations:

$$\max_{\alpha} \ln ML \to \alpha_{\rm opt}, \qquad \max_{\beta} \ln ML \to \beta_{\rm opt}.$$
(6)

Similarly as in (5), the appropriate formulas for computations of hyperparameters can be found in the above quoted literature.

Formulas (6) are of recurrent type and need estimation of initial values  $\alpha_{in}$  and  $\beta_{in}$ . In the book by Nabney [37] the procedure EVIDENCE was given for the computation of parameter  $H_{opt}$  and hyperparameters  $\alpha_{opt}$  and  $\beta_{opt}$ .

# 4.1.4. Gaussian Process model

Besides SBNN the Gaussian Process model (GP) is worth mentioning. GP is related to the interpolation of Radial Basis Function NN, in which centers of BFs are placed at input pattern points. In case of GP the kernels function k ( $\mathbf{x}^{p}, \mathbf{x}^{r}$ ) is applied instead of classical RBFs, see [2] and supplementary paper [59] written by Słoński. The predictive distribution of mean in the Gaussian probability density for a new input vector  $\mathbf{x}^{p+1}$  is  $y(\mathbf{x}^{p+1}) = \mathbf{k}^{\mathrm{T}} \mathbf{C}^{-1} \mathbf{k}$ . The covariant matrix  $\mathbf{C}_{(P \times P)}$  is composed of components  $C_{pr} = k(\mathbf{x}^{p}, \mathbf{x}^{r}) + (1/\sigma^{2})$ , where the standard error  $\sigma$  of the input set was introduced, cf. [2]. In the group of kernel methods the above sketched approach is called Gaussian Process Model, see [2], the GP-BNN in [67].

GPM is formulated without a vector of weights **w**. Thus, the main computational effort is related to the inversion of the covariant matrix, to have  $\mathbf{C}_{(P \times P)}^{-1}$ , since the inversion of matrix **C** can be computationally very costly in case of a large number of P.

# 4.1.5. Applications of Bayesian neural networks in problems analyzed after 2005

Bayesian Neural Networks (BNNS) were applied to a number of problems analyzed by RGANN and discussed at SemANN sessions. The basic questions were related to the accuracy and numerical efficiency vs. those provided by standard ANNs. In the-state-of-the-art paper [67], the attention was turned to formulation of different Bayesian networks and methods, especially to so-called Semi Bayesian NN (SBNN). Also the Gaussian Process model (GP) is worth emphasizing, see [59], because of its simplicity and accuracy. Another general remark is that it was numerically proved that the True BNN (TBNN) networks can be numerically costly in the sense of a very high CPU times.

The general conclusions expressed above were verified on examples of BNNs applications to the analysis of several problems of mechanics. At the beginning, the simulations of displacement response spectra of buildings subjected to paraseismic excitations were analyzed and reported in [64, 67]. Then a great deal of attention was paid to selected problems of concrete mechanics. Prediction of the strength of high performance concrete and normal concrete fatigue failure were investigated by Słoński by means of various types of Bayesian neural networks and methods, see [55–59]. The Bayesian methods were also applied by Krok in paper [20] to the numerical prediction of the hysteresis loops of compressed concrete.

The True BNN was applied by Kaczmarczyk and Waszczyszyn in paper [15] to the identification of a material internal parameter and evaluation of sigma-bar region for this parameter.

SBNNs are now a standard tool for neurocomputing analyses carried out at the Rzeszów TU, Poland. SBNNs were applied in hybrid computational systems developed for structure health monitoring, see [32], and a number of identification problems related to structural mechanics and soil compaction, see papers [21, 22].

# 4.2. Data processing and development of ANNs

#### 4.2.1. Principle Component Analysis

In Subsubsec. 3.4.3 the application of replicator to the reduction of input space dimensionality was discussed. The replicator was formulated as an autoassociated neural network of architecture MLP: D-d-D applied to the mapping  $\mathbf{x}_{(D\times 1)} \to \widehat{\mathbf{x}}_{(D\times 1)} = \mathbf{x} - \boldsymbol{\varepsilon}$ , see Fig. 1.

Instead of application of replicator we can use the Principle Component Analysis (PCA). We start from formulation of the pattern  $\mathbf{x}_{(D\times 1)}^{p}$  correlation matrix  $\mathbf{S}_{(D\times D)} = \sum_{p=1}^{P} \mathbf{x}^{p} (\mathbf{x}^{p})^{\mathrm{T}}/P$ . Then eigenvalues and eigenvectors of  $\mathbf{S}$  are computed  $(\lambda_{j}, \mathbf{q}_{j})$ . The eigenvalues are arranged in a decreasing order  $\lambda_{1} \geq \lambda_{2} \ldots \geq \lambda_{d} \ldots \geq \lambda_{D}$  and the *d*th highest value defines the compressed dimension of the input subspace  $\mathcal{R}^{d} < \mathcal{R}^{D}$ . It can be easily proved that the vector of principle components can be explicitly written as  $\boldsymbol{\xi}_{(d+1)} = \mathbf{Q}_{(d\times D)}^{\mathrm{T}} \mathbf{x}_{(d+1)}$  and the reconstruction vector is  $\mathbf{x} = \mathbf{Q}\boldsymbol{\xi}$ . The approximation error vector can also be explicitly computed as  $E = \sum_{j=d+1}^{D} \xi_{j} q_{j}$ .

The above written a short resumé enables us to highlight the superiority of the PCA over the application of replicator neural network. The first advantage is the explicit formula for estimating the number of conserving eigenvalues d of the correlation matrix, which defines the number of compressed inputs. Without any additional learning procedures, corresponding to the replicator, we can explicitly compute the truncation error E. Due to storing of the eigenpairs  $(\lambda_c, q_c)_{c=1}^d$  we can explicitly reconstruct components of the inputs  $x_j^p$ .

The advantages listed above caused the replicator to be changed into the PCA. Such a way of reducing the input space dimensionality has been shown in the current literature, see e.g. [2]. In 2006 the PCA was applied in paper [25] written by Kuźniar and Waszczyszyn. In the paper PCA was applied in a simple problem of prediction of the basic eigenfrequencies of small buildings.

An original application of PCA was proposed by Kaczmarczyk and Waszczyszyn in paper [15]. An internal material parameter at the microlevel was identified and by means of PCA where the transition for the unidimensional input space d = 1 was possible. It enabled us to apply efficiently the True BNN.

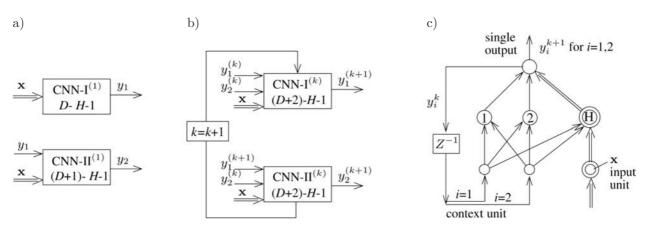
It was also shown in paper [22], written by Kłos and Waszczyszyn that the PCA transformation of ten inputs gave a satisfying improvement of the network SBNN accuracy for the identified compaction parameters without a compression of the input data.

#### 4.2.2. Recurrent Cascade Neural Network (RCNN)

In literature the Cascade Neural Network is known as a modification which changes the network MLP: D-H-M into M cascade of networks CNN-l with single outputs at levels l = I, II, ..., M, cf. [68]. In Fig. 2a the two networks are shown with single outputs in 2D space, i.e. M = 2 and CNN-I<sup>(1)</sup>: D-H- $y_1^{(1)}$  and CNN-II<sup>(1)</sup>:  $(D + y_1^{(1)})$ -H- $y_2^{(1)}$ . In paper [22] this set of networks was called Standard CNN, corresponding, in fact, to the first cycle or recurrence k = 1. Figure 2b illustrates an extension of the SCNN, called in [22] Recurrent CNN. For the sake of simplicity only two outputs are considered. It is valid for the recurrence cycles k > 1 so that at both levels the networks are of architecture (D + 2)-H-1.

The network RCNN corresponds to Jordan type networks with single outputs, cf. [48], see Fig. 2c. The network is composed of the context unit and input unit. The context unit has only inputs with self feedback outputs. Thus, the behavior of the context unit corresponds to the recurrent type network. For prediction (testing) process the RCNN (and, especially, the context unit) depends on finding stable solutions for fixed values of weights, computed in the frame of the supervised learning.

The above mentioned problems are discussed by Piątkowski and Waszczyszyn in the supplementary paper [50], where the control parameters correspond to the identified locations of a mass attachment to a plate. In [50] it was stated that because of self feedbacks the recurrence itera-



**Fig. 2.** a) Cascade Neural Network (CNN) for the recurrence cycle k = 1, b) Recurrent Cascade Neural Network (RCNN) for k > 1, c) RCNN as a modification of Jordan neural network.

tion process can be inaccurate or even divergent. This defect of the algorithm has been partially overcome by the introduction of a barrier bound.

The investigated problem turned out to be much more complicated than that analyzed in [22]. The identification of three geometrical parameters of circular arches needed only K = 3 iteration cycles for satisfactory identification of the arch parameters. The recurrent iteration was convergent with the expected values also for artificial noises added to eigenfrequencies, computed by FEM as the arches dynamic responses. In case of identification of the localizations of mass attachment by means of measurement data, investigated in [50], the recurrent iteration needed up to K = 20 recurrence. The other problem was that the testing process is related to the non-supervised learning and the introduction of the barrier bound. Such an approach needs to be further investigated in order to fulfill all conditions of objective identification.

#### 4.3. ANNs in geotechnical problems

In the supplementary paper [54] Sulewska presented a great interest in ANNs applications in geotechnical problems. She pointed out the following six selected problems: 1) prediction of the Overconsolidation Ratio, 2) estimation of potential soil liquefaction, 3) prediction of foundation settlement, 4) evaluation of piles bearing capacity, 5) prediction of cohesive parameter for cohesive soils, 6) compaction built of cohesionless soils.

After 2005 the problems of soil compaction were extensively developed by Sulewska in her D.Sc. monograph [53]. She applied MLP neural networks for prediction and identification of compaction parameters for cohensionless soils. The investigations were based on measurements carried out by means of the Dynamic Probe Light (DPL) tests. The superiority of the MLP applications over the standard statistical methods has been numerically proved with respect to applications of DPL and MLPs in the surface soil layers. New results were also obtained applying MLPs to the identification of compaction parameters basing on the Proctors test.

The last mentioned problem was investigated by Kłos, Sulewska and Waszczyszyn and new results are presented in the supplementary paper [21]. Semi Bayesian NNs were applied to the identification of compaction characteristics basing on the Proctors tests for postglacial sands of North-East Poland. The hybrid system EMP&SBNN was formulated. Besides application of the Bayesian methods mentioned in Subsubsec. 4.1.3, the PCA method was also applied in input data preprocessing. It was stated that the application of the PC transformation can improve the accuracy of BSNN networks without performing data compression.

The supplementary paper by Wojciechowski [71] concerns the development of a hybrid approach to the problem of soil parametric identification in deep excavation. This problem, originated in Wojciechowski's Ph.D. Thesis [70], was related to the application pf ANNs for quick simulation of numerical patterns in direct analysis. The simulation of selected response displacements, as functions of the identified soil parameters, was used in a mathematical programming method for optimal identification of these parameters.

Once more, the application of the hybrid computational system FEM/MNN/p-EMP by Pabisek in her supplementary paper is worth mentioning [44]. In the system a neural procedure for the identification of an equivalent material model for grain soils was adopted in the form of the Material Neural Network model (MNN). The development of a hybrid system and introduction of measurement carried out 'in situ' on real soils might open the door to new non-destructive methods for modeling real soils without costly laboratory tests.

# 4.4. ANNs in geodesy

Mrówczyńska's Ph.D. Thesis [34] deals with new possibilities of ANNs in geodesy. As a successor of Prof. J. Gil, see [9], she has applied different ANNs in the analysis of various problems of geodesy. In the supplementary paper [35], she applied the Suzuki-Sugeno-Kang fuzzy system, then MLP, Radial Basis Function NN and RCNN to the analysis of the following three problems: 1) digital approximation of terrain problems, 2) transformation of the Polish system coordinates "1965" into the official system "2000", 3) prediction of time series for point displacements using the GPS-RTK technique.

# 4.5. Advisory system for industrial floors repairs

In the supplementary paper [6] Gajzler formulated an expert system for classification of defects and application of appropriate repairs of industrial floors. An original idea was introduced into the system concerning the application of modified MLPs in the interference engine to classification purposes of formulation rules in the knowledge base.

# 4.6. Promotion of ANNs after 2005

Papers with recent new research results obtained by RGANN members were presented at a number of Polish and foreign scientific conferences and congresses. The main topics of the corresponding papers are reflected in the supplementary papers published in this Special Issue of CAMES. Such papers seem to be an important contribution for promotion of ANNs as a new tool for analyzing of complex engineering problems. The minisymposia organized by the members of RGANN should also be mentioned. After 2005 five minisymposia, devoted to ANNs and soft computing methods were organized at ECCC 2006 in Lisbon, Portugal; CCM 2009, Zielona Góra, Poland; ECCM 2010, Paris, France; and CMM2011, Warsaw, Poland.

All the members of RGANN from the Rzeszów UT, Poland, participated in the organization of three scientific conferences in Poland. These conferences were: ECCOMAS Thematic Conferences on the Inverse Problems of Mechanics IPM2009 and IPM2011 in Łańcut and Sieniawa, see [75, 76] and the 57th Conference on Civil Engineering, Krynica 2011, see[74]. At this conference the Special Session devoted to ANN applications in CE took place and the presented papers are now published in the CAMES Special Issue. Altogether 29 papers (fifteen were prepared for the Krynica Special Session) were presented at these conferences by the members of the RGANN group.

In the final part of the paper let me mention the promotion activity of ANNs by the participants in very prestigious lectures delivered at the Centre International des Sciences Mécaniques in Udine, Italy. In 2007 the Advanced CISM School on Soft Computing was organized by Waszczyszyn who together with Słoński delivered a series of lectures [67], published in the book [62] under Waszczyszyn's edition. In this way the teaching activity, originated in Udine in 1998 and continued in 2003, see books [36, 62], have continued the promotion of ANNs.

# 5. FINAL REMARKS AND SOME CONCLUSIONS ON RESEARCH AND TEACHING ACTIVITIES ON ANN APPLICATIONS IN CE AFTER 2005

- 1. Two D.Sc.E. monographs and eight Ph.D. Theses were "defended".
- 2. From among new results in the area of ANNs development the following research are worth emphasizing:
  - applications of Bayesian Neural Networks and Bayesian methods in design and learning of neural networks;
  - Recurrent Cascade Neural Network, as a new network with promising features for identification analysis, needs further research;
  - Principal Component Transformation and PCA (PC Analysis) can be applied not only for compression of input data but also for improving the accuracy of neural approximations.
- 3. ANNs have been successfully used in new areas of their applications, i.e. in geotechnics and geodesy.
- 4. Research on combining ANNs with measurement techniques are under successive development at the Rzeszów UT. The main targets of this activity are new non-destructive methods in the area of structure health monitoring, diagnosis and repair.
- 5. The dominating effort has been devoted to development of hybrid computational systems and their applications in different CE problems. The system FEM/CNN/EMP of very high degree of component fusion is worth emphasizing. Formulation of other systems FEM&ANN and EMP& RCNN of low degree of fusion leads to numerically efficient systems for simulation and identification of special problems of CE.
- 6. In the years 2006-2011 a number of activities have been continued. Besides the teaching of young researches, e.g. an Advanced CISM Course organized in Udine, Italy, in 2007 also new scientific conferences were organized after 2005, i.e. ECCOMAS Thematic Conferences on Inverse Problems of Mechanics IPM 2009 and IPM 2011 in Łańcut and Sieniawa near Rzeszów, Poland.

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